Let me start today's discussion with a question from our Arithmetic book. I love this question because it is very crafty (much like actual GMAT questions, I assure you!) It looks like a calculation intensive question and makes you spend 3-4 minutes (scribbling furiously) but is actually pretty straight forward when understood from the 'weighted average' perspective. We looked at an easier version of this question in the last post.

John and Ingrid pay 30% and 40% tax annually, respectively. If John makes $56000 and Ingrid makes $72000, what is their combined tax rate?

a. 32%

b. 34.4%

c. 35%

d. 35.6%

e. 36.4%

If we do not use weighted averages concept, this question would involve a tricky calculation. Something on the lines of:

Total Tax = (30/100)*56000 + (40/100)*72000

Tax Rate = Total Tax / (56000 + 72000)

But we know better! The big numbers – 56000 and 72000 are just a smokescreen. I could have as well given you $86380 and $111060 as their salaries; I would have still obtained the same average tax rate! What is important is not the actual values of the salaries but the relation between the values i.e. the ratio of their salaries. Let me show you.

We need to find their average tax rate. Since their salaries are different, the average tax rate is not (30 + 40)/ 2. We need to find the 'weighted average of their tax rates'. In the last post, we discussed

w1/w2 = (A2 – Aavg) / (Aavg – A1)

The ratio of their salaries w1/w2 = 56000 / 72000 = 7/9

7/9 = (40 – Tavg) / (Tavg – 30)

Tavg = 35.6%

Imagine that! No long calculations! In the last post, when we wanted to find the average age of boys and girls – 10 boys with an average age of 17 yrs and 20 girls with an average age of 20 yrs, all we needed was the relative weights (relative number of people) in the two groups i.e. 1:2. It didn't matter whether there were 10 boys and 20 girls or 100 boys and 200 girls. It's exactly the same concept here. It doesn't matter what the actual salaries are. We just need to find the ratio of the salaries.

Also notice that the two tax rates are 30% and 40%. The average tax rate is 35.6% i.e. closer to 40% than to 30%. Doesn't it make sense? Since the salary of Ingrid is $72,000, that is, more than salary of John, her tax rate of 40% 'pulls' the average toward itself. In other words, Ingrid's tax rate has more 'weight' than John's. Hence the average shifts from 35% to 35.6% i.e. toward Ingrid's tax rate of 40%.

Let's now look at PS question no. 148 from the Official Guide which is a beautiful example of the use of weighted averages.

*If a, b and c are positive numbers such that [a/(a+b)]*20 + [b/(a+b)]*40 = c and if a < b, which of the following could be the value of c?*

(A) 20

(B) 24

(C) 30

(D) 36

(E) 40

Let me tell you, it isn't an easy question (and the explanation given in the OG makes my head spin).

First of all, notice that the question says: 'could be the value of c' not 'is the value of c' which means there isn't a unique value of c. 'c' could take multiple values and one of those is given in the options. Secondly, we are given that a < b. Now how does that figure in our scheme of things? It is not an equation so we certainly cannot use it to solve for c. If you look closely, you will notice that the given equation is

(20*a + 40*b) / (a + b) = c

Does it remind you of something? It should, considering that we are doing weighted averages right now! Isn't it very similar to the weighted average formula we saw in the last post?

(A1*w1 + A2*w2) / (w1 + w2) = Weighted Average

So basically, c is just the weighted average of 20 and 40 with a and b as weights. Since a < b, weightage given to 20 is less than the weightage given to 40 which implies that the average will be pulled closer to 40 than to 20. So the average will most certainly be greater than 30, which is right in the middle of 40 and 20, but will be less than 40. There is only one such number, 36, in the options. ‘c’ can take the value ‘36’ and hence, (D) will be the answer. Elementary, isn’t it? Not really! If you do not consider it from the weighted average perspective, this question can torture you for hours.
These are just a couple of many applications of weighted average. Next week, we will review Mixtures, another topic in which weighted averages are a lifesaver!